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Fracture Nucleation from a Compression-parallel, Finite-width Elliptical Flaw

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INTRODUCTION

How tensile fractures form and propagate in response to crack-parallel compression has been puzzling the rock mechanics community for decades. The direct application of engineering fracture mechanics to the problem of tensile fracture in a compressive environment appears to be invalid, because the zero-width mathematical crack model is unresponsive to the normal stress that is coaxial to the crack direction. Fractures can however be observed to propagate parallel to a compressive stress, and pre-existing compression-parallel cracks are known to extend in their own plane when subjected to increasing compressive stress [1, 2].

Fracture mechanics needs tensile stresses to nucleate tensile fractures. Theoretical models exploit the inhomogeneity of rock materials (flaws) to produce localized tensile stresses in an otherwise compressive stress environment. The crack starting flaws can be pre-existing microcracks, grain boundaries, elastic moduli mismatches, cylindrical pores, dislocation pile-ups and Herztian contacts [1, 3]. Perhaps, the best example for this approach is the sliding-crack model of Nemmat-Nasser [4, 5]. The fracture nucleating flaw is a "sliding crack", a closed flat ellipse that is inclined to the compression direction. The tension is generated at the crack tip through the shear stress along the plane of the crack and through the normal stress that is perpendicular to it (the crack opening and the in-plane sliding modes of fracture mechanics). The stress intensity factor formulation of the mathematical crack model neglects the other normal stresses and in particular the one that lies parallel with the major axis of the ellipse. The nucleated cracks (wing-cracks) propagate along the gently curving maximum principal stress trajectory until the crack becomes parallel with the far-field (applied) compressive stress. The fracture process stalls at this point.

Practically, all the numerical simulations that use inhomogeneities show that the extension fracture arising from an inhomogeneity is comparable in size to the inhomogeneity itself. Once, the newly generated fracture propagates beyond the zone of influence of the inhomogeneity, tension yields to compression and the fracture process halts. The sliding crack or any other stress starter should therefore be observable under the microscope. Microscopic observations, however, report only the presence of compression parallel extension fractures, and only rarely can the actual crack starter be identified [1, 2, 3]. Since, the tensile fractures aligned with the compression direction extend in their own plane with increasing compression, they themselves must be considered as potential crack starters. The zero-width, mathematical crack of fracture mechanics is clearly unsuitable for this purpose.

Sensitivity to crack-parallel compression can be introduced by replacing the zero-width, mathematical crack with a general elliptical crack, with the major axis representing the crack length oriented along the maximum principal stress trajectory, and the minor axis representing the crack width. We call this the FInite-Width Elliptical Crack, or FIWEC for short. Using the FIWEC model in connection with some type of fracture criterion, the propagation of compression-parallel tensile cracks can be modelled as a series of crack nucleation events from an open, finite-width elliptical flaw of progressively increasing length.

THE FINITE-WIDTH ELLIPTICAL CRACK MODEL

Two subjects are keenly associated with the fracture problem: the shape and the size effect. In modelling the fracture problem, the shape effect is addressed by introducing an idealized fracture shape, whereas the size effect is left to the fracture criterion. For example, fracture mechanics simplifies the stress concentrator into a zero-width flat elliptical cavity, the mathematical crack. The size effect is "naturally" included through fracture criteria, which in turn is expressed through either the stress intensity factor formulation or through a stress averaging scheme [7, 8].

Neglecting the crack width in a crack model has tremendous mathematical advantages. However, in doing so the sensitivity of the mathematical crack model to normal stress that is coaxial with the major axis of the elliptical crack is sacrificed. As a consequence, the mathematical crack will not extend in response to the normal stress acting in the plane of the crack. Since in

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practice, tensile fractures are known to extend in response to increasing maximum principal stress, the mathematical crack is ill-suited to model the fracture process in the compressive environment of underground mining. The FIWEC model, on the other hand, retains the crack width by representing the crack with an ellipse of a general shape (Figure 1).

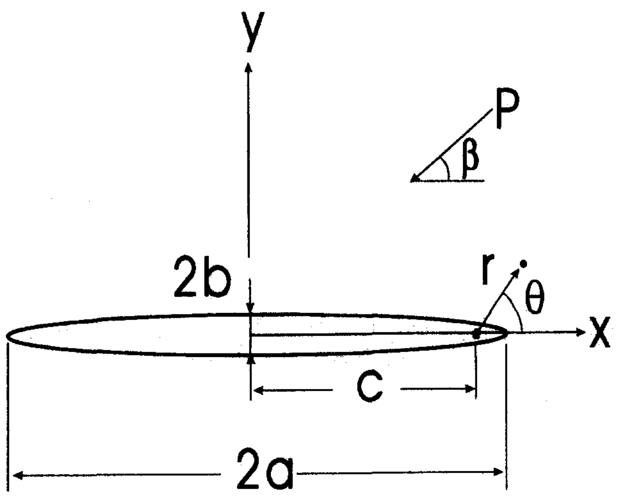

Figure 1. The geometry of the Finite-width Elliptical Crack Model

Stress intensity factor for the FIWEC crack

Whether one uses engineering fracture mechanics or a stress-averaging scheme [7, 8], fracture nucleation is controlled by the stress field around the crack tip. A number of solutions exist that describe the stress field around an elliptical cavity [9]. The mathematical procedure begins with the stress functions for an elliptical

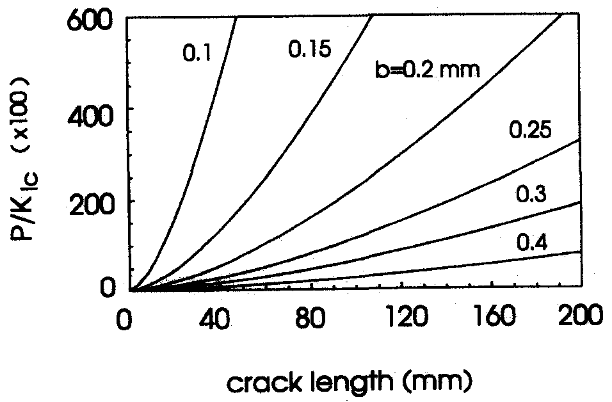

Figure 2. Crack initiation from a finite-width elliptical crack; b is the half-width of the ellipse.

hole. The terms that include the radial distance from the focal point are expanded into power series. The asymptotic variation of the stresses at the crack tip are then obtained by neglecting the higher order terms. The elastic asymptotic stress field around a FIWEC crack tip consists of two singular terms:

$$\frac{\sigma_{ij}}{P}=f_{ij3}(\theta,e,\beta)\left(\sqrt{\frac{2c}{r}}\right)^3+f_{ij1}(\theta,e,\beta)\sqrt{\frac{2c}{r}}+f_{ij0}(\theta,e,\beta) \quad (1)$$

Here the external compressive load, P, acts at an angle β to the crack axis; e is the aspect ratio of the ellipse having minor axis b and major axis a (Figure 1). C is the focal length of the ellipse. The mathematical expressions for the coefficients of f_{ij3}, f_{ij1} and f_{ij0} are rather complex. For the stress component σ_{yy}, the three coefficients are:

$$f_{yy3}=-\frac{1}{8}\left(1-e^{2\xi_0}\right)^2\left[\cos\left(2\beta-\frac{3\theta}{2}\right)-e^{-2\xi_0}\cos\frac{3\theta}{2}\right]$$

$$f_{yy1}=-\frac{1}{4}\cos\left(2\beta+\frac{\theta}{2}\right)-\frac{1}{64}\left(1+38e^{2\xi_0}-15e^{4\xi_0}\right)\cos\left(2\beta-\frac{\theta}{2}\right)$$

$$+\frac{1}{64}\left(e^{2\xi_0}+38+e^{-2\xi_0}\right)\cos\frac{\theta}{2}-\frac{1}{8}\cos\frac{5\theta}{2}+\frac{1}{8}e^{2\xi_0}\cos\left(2\beta-\frac{5\theta}{2}\right)$$

$$f_{yy0}=-\frac{1}{4}\left(1-e^{2\xi_0}\right)^2\cos 2\beta \quad (2)$$

Letting the crack width equal zero, (1) and (2) reduce to the well known stress equations of fracture mechanics.

For the case of uniaxial compression along the crack axis (β = 0), the transverse tensile stress at the crack tip is determined by:

$$\sigma_y \approx -\frac{K_I}{\left(\sqrt{2\pi r}\right)^3}\cos\frac{3\theta}{2} \quad (3)$$

K_I is the opening-mode stress intensity factor that represents the effect of the crack-parallel compressive stress:

$$K_I=\left(e\sqrt{\pi a}\right)^3 P+O(e^4) \quad (4)$$

O stands for additional terms that are usually negligible. If $b = 0$ then $e = 0$ and K_I vanishes; the solution reduces to the case of the zero-width, mathematical crack model. When $K_I = K_{Ic}$, the crack extends according to:

$$a=\frac{\pi b^2 P^{2/3}}{K_{Ic}^{2/3}} \quad (5)$$

This is shown graphically in Figure the expected "hardening"; with i greater and greater increments of stress are needed to