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Tensile fracture from circular cavities loaded in compression

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Abstract. When a block of rock containing an equi-dimensional void is loaded in compression, the resulting fracture may form at one of three basic positions: at the tensile stress concentration of the perimeter (primary fracture), at the compressive stress concentration of the perimeter (slabbing fracture), or off the perimeter, remote to the cavity (remote fracture). All three are genetically similar; they form and propagate parallel to the direction of the maximum compressive stress. The location of the fracture with respect to the cavity is controlled by the cavity size and the confining pressure.

Although LEFM solutions exist for the primary fracture, the mathematical crack of fracture mechanics is ill-suited to analyze fractures that form in a primarily compressive state of stress (remote and slabbing fractures); the mathematical crack is independent of the compressive stress acting along its plane. A stress-based solution is proposed that incorporates the effect of both the maximum and the minimum principal stress. The major shortcoming of conventional stress-based techniques, the lack of size dependence, is removed by a procedure of stress averaging over a constant distance or area. For the case of the cylindrical cavity, stress averaging along the primary fracture path can be built into a closed-form solution. Averaging stresses over a constant area requires numerical techniques.

Physical experiments, involving the compression loading of cylindrical cavities in three rocks: a granite, a limestone and a salt rock provide data for the comparison and the calibration of the theoretical criteria. Stress averaging over a constant area gave the best agreement with the test data.

1. Introduction

Since Griffith [1] laid the theoretical foundation for tensile cracking in compressive states of stress, the possible fracture mechanism and its analytical or numerical model has received considerable attention. Most of this attention focussed on the concept of the starting crack and the method of its analysis. Griffith and workers in present-day fracture mechanics nucleate fractures from elliptical cavities with a large aspect ratio (the flat crack). However, tensile fracture initiation from more equi-dimensional cavities is also important when interpreting fracture in porous materials (e.g. [2]), or around deep boreholes (e.g. [3]), tunnels or other mining related underground cavities.

According to the position with respect to the equi-dimensional cavity, three types of tensile fractures have been noted in physical modes [4]. First, there are fractures that grow out of the tensile, tangential stress concentrations of the perimeter and intersect the cavity wall (primary fracture). The second type is associated with the compressive tangential stress concentrations of the perimeter where the fractures run parallel to the perimeter (slabbing fracture). The third group forms off the perimeter, at positions where a critical combination of tensile and compressive stresses create the condition necessary for fracture (remote fracture). There is no genetic difference between the three fracture types. They all propagate along the maximum principal stress (compression positive) trajectory, or more precisely, run perpendicular to the minimum principal stress [5]. There is no shear displacement along the plane of the crack, only dilation perpendicular to it. Therefore the primary, the remote and the slabbing fractures are all tensile fractures. In a low confining stress environment and with a rising far-field, maximum principal stress (axial load), all three types may eventually appear with the primary fractures forming first, the remotes second, and the slabbing fractures last. When the confining pressure is higher, as in the case of boreholes, the slabbing fracture mode may be the only one active.

Theoretical modeling of fracture around cavities may follow the traditional stress (safety factor) approach (e.g. [6]), or the more fashionable fracture mechanics formulation. Both techniques have serious shortcomings. The stress approach cannot account for the considerable size effect, especially for small cavities (pores), while fracture mechanics is ill-suited to interpret the remote and slabbing types of fractures [7]. In this paper, the stress-based approach is developed further by introducing the closed-form linear-averaging and the numerical area-averaging techniques; both utilize multiaxial, stress-based fracture criteria. The theoretical modeling will be supported through calibration from physical tests using blocks of rocks having a circular cavity of variable radii. Three rock types have been used, a very brittle and strong granite (Lac du Bonnet granite), a semi-brittle limestone (Tyndallstone), and a semi-ductile salt rock (Lanigan and Vanscoy potash).

2. Theoretical models

The presence of a cavity in an otherwise homogeneous mass of rock causes the state of stress to vary from point to point. To evaluate the state of fracture at a specific point around a circular cavity, one must first define the state of stress and then compare it with the fracture resistance of the material located at the point. The comparison is made through empirical fracture criteria. The simplest method is based on direct stress parameters, often combined in a safety factor. To compute the safety factor for underground excavations, Hoek and Brown [6] compare the indicated maximum principal stress at a point (σ_1) with the strength at the point, defined as the maximum principal stress at failure (σ_{1f})

$$SF = \frac{\sigma_{1f}}{\sigma_1} = \frac{\sigma_3 + \sqrt{\sigma_3 m\sigma_c + s\sigma_c^2}}{\sigma_1}. \tag{1}$$

The strength is modeled through a $y^2 = f(x)$ type of parabola that is a function of the minimum principal stress at the point, but is independent of the intermediate principal stress. In the definition of the safety factor, σ_3 and σ_1 are the minimum and the maximum principal stresses at the point in question, σ_c is the uniaxial compressive strength, while m and s are the fitting and the rock mass scaling parameters respectively. For the intact rock specimen of this study $s = 1$. m is a constant that depends on the rock properties and can be determined by fitting (1) to triaxial test data. This paper follows the geotechnical convention in assigning the positive sign to compressive stress and compressive strength, while tensile stress and strength become negative quantities.

The Hoek and Brown formulation for strength gives a good fit to triaxial compression data obtained at high confining pressure; it is however rather inaccurate in the low compression to tension field of loading. An alternative, the Rocker function [8] has been designed to anchor the strength curve at two points, representing the uniaxial compressive and the uniaxial tensile strength

$$\sigma_{1f} = \sigma_c\left(1 - \frac{\sigma_3}{\sigma_t}\right)^R.$$