Numerical modeling of ground deformation and source mechanism related to co-seismic change in gravitational potential

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Abstract


The release of gravitational potential energy is implicit in ground surface deformation, as was seen in the 1954 Fairview Peak, 1964 Alaska and 1976 Tangshan earthquakes where more energy was released by surface subsidence than was radiated as seismic waves.

Based on the numerical results of dip and thrust faulting driven by either gravitational tectonic stress alone or additional external forces, the elastic stress accumulation process in the front of the fault tip is required, as is usual, prior to any faulting slip, but the energy release would be mainly derived from the downward movement of the hanging wall of a normal fault or the footwall of a thrust fault. In other words, there will be a decrease in gravitational potential energy in the upper part of the subsiding wall to generate seismic waves, and an increase in elastic strain energy in the lower part either to cause after shocks or to bring about cessation of further fault movement. In addition, by geodetic surveying along a fault prior to an earthquake opportunities exist to judge the nature of the tectonic force and the damage zonation.

Introduction

During an earthquake an enormous amount of energy is released in the form of seismic waves and abrupt energy adjustment takes place within the focal body, where the gravitational energy of one subregion is transformed into the elastic strain energy in another, and vice versa. Most researchers emphasize the elastic strain energy change during the earthquake process and almost all previous studies of the focal mechanism have taken no account of the gravitational force. This simplification is reasonable for faults with large horizontal movement. However, there are many earthquakes with large vertical displacements such as the 1964 Alaska and 1976 Tangshan earthquakes. Some theoretical and practical studies have shown that an important energy source for seismic waves is the released gravitational energy in the earthquake focus body (Dalhen, 1977; Savage and Walsh, 1978; Barrows, 1983). Here, we will study another means of analyzing the characteristics of spatial energy change within the focal body during an earthquake. The fault is treated as a crack in a fracture mechanics approach and the energy change due to gravity alone or additional external forces is calculated using the finite element method.

Details of computation

Quantitative analysis

When a crack acts as a fault, the crack surfaces are in contact by frictional force. They cannot indent into each other in the normal direction and
both the normal stress \( F_n \) and the tangential stress \( F_t \) must satisfy the frictional slip law. In this situation the methods of general engineering fracture mechanics are not valid for solving the present crack problem. To overcome this difficulty an improved finite element formulation was derived from the variation principle with Lagrange constraint multipliers, and the crack surface is allocated double nodes and discretized into contact elements. Furthermore, a special crack tip contact element is developed to take into account the crack tip stress singularity. Comparison of the numerical calculation with analytical solutions shows that the programme so designed has achieved a satisfactory precision. This will be introduced in detail elsewhere.

The Mohr–Coulomb principle is chosen as the frictional slip law for the crack surface:

\[
F_t = C_0 + \mu \times F_n
\]

(1)

\( C_0 \) and \( \mu \) are the cohesion and frictional coefficient on the surface respectively, \( C_0 \) is set to zero in this study.

The energy density variation and its physical significance

For any elementary volume \( dV \) in a deformable solid the variation in its gravitational energy density \( (dE_g/dV) \) and strain energy density \( (dE_s/dV) \) before and after rupture is:

\[
dE_g/dV = \rho \times g \times \Delta U_z
\]

(2)

and

\[
dE_s/dV = \left( \sigma^T \epsilon - \sigma_0^T \epsilon_0 \right)/2
\]

(3)

where \( \rho \) is mass, \( g \) is acceleration due to gravity, \( \Delta U_z \) is the increment of displacement in the gravity direction. \( \sigma_0 \), \( \epsilon_0 \) and \( \sigma \), \( \epsilon \) are the stress and strain tensors before and after rupture respectively, and the superscript \( T \) represents the transpose of the tensor. When \( dE_g/dV > 0 \), the volume will release gravitational potential or otherwise absorb energy to elevate its gravitational potential. Negative \( dE_g/dV \) signifies that the elastic strain energy will be released or otherwise absorbed.

The difference between \( dE_g/dV \) and \( dE_s/dV \) is the energy density variation \( (dE/dV) \):

\[
dE/dV = dE_g/dV - dE_s/dV
\]

(4)

The integration of \( dE/dV \) over a region \( \Omega \) gives the total energy release \( (\Delta E) \):

\[
\Delta E = \int \left( dE/dV \right) \times d\Omega
\]

(5)

During an earthquake, both the released gravitational potential and the elastic strain energy \( (\Delta E) \) of different regions in the focal body are transformed into the radiated energy of the seismic waves. \( \Delta E \) can be obtained by means of quantitative quadrature, such as a first-order Gaussian integration:

\[
\Delta E = \sum_{i=1}^{n} (dE/dV)_i \times (\Delta \Omega)_i
\]

(6)

\( (dE/dV)_i \) is the \( i \)-th element \( dE/dV \) value at the Gaussian integration point, \( (\Delta \Omega)_i \) = volume, and \( n \) = the total number of elements in the region.

The results for energy and displacement are standardized with a characteristic length, the pre-existing fault length \( a \). Thus:

\[
(dE/dV)_s = (dE/dV)_0 \times a^2
\]

(7)

\( (\Delta E)_s = (\Delta E)_0/a \)

(8)

\( (U)_s = (U)_0/a \)

(9)

The subscripts “0” and “s” represent before and after standardization. In this scheme, the unit of standardized energy is the kilogramme but there is no unit for displacement. For simplicity the values shown in the figures and table are divided by \( 10^{-4} \).

Numerical model

It is to be emphasized here that we are analyzing the effect of gravity during an earthquake, and the model is simplified to a two-dimensional plane strain quasi-static elastic deformation model. Further, the energy change before and after fault slip is only discussed conceptually, without dynamic simulation.

Consider a listric-like crack in the crust with uniform mass distribution. The length